## c-Chart … 1/2



**Example 4**:

A teacher wishes to monitor and control the class attendance. He records the number of absentees over an academic week and the result is Mon: 5, Tue:3, Wed:3, Thu:2, Fri:4. The number of students on roll on these five days remained 40. Develop a control chart.

Explanation: This is a case of defects (absentees) in a unit (class). The sample size (class strength is fixed). ∴ Correct Chart is **c**

| Day | Class Strength i.e. Sample Size (n) | Number of Absentees i.e. Defaults/ Defects (d) |
|---|---|---|
| Mon | 40 | 6 |
| Tue | 40 | 4 |
| Wed | 40 | 5 |
| Thu | 40 | 3 |
| Fri | 40 | 5 |

**Cont Limits = $\bar{c} \pm z\sqrt{\bar{c}}$**

where
$\bar{c}$ = CL & mean number of defects
$z$ = Quality standard, eg 3 sigma or more, or degree of confidence

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## c-Chart … 2/2



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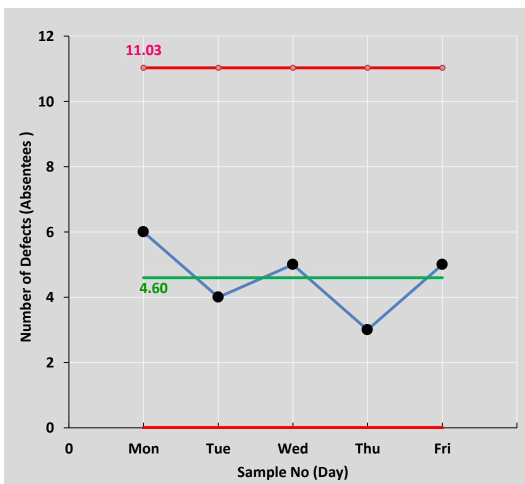
| Day | Sample Size (n) | Defaults/ Defects (d) | Prop Defaults (d/n) | LCL | UCL |
|---|---|---|---|---|---|
| M | 40 | 6 | Not Reqd | -1.83 | 11.03 |
| T | 40 | 4 | | -1.83 | 11.03 |
| W | 40 | 5 | | -1.83 | 11.03 |
| Th | 40 | 3 | | -1.83 | 11.03 |
| F | 40 | 5 | | -1.83 | 11.03 |
| ∑ | 200 | 23 | | | |
| $\bar{c}$ | 23/5 = 4.6 absentees per day/class | | | | |




**Cont Limits = $\bar{c} \pm z\sqrt{\bar{c}}$**

Here, the SD $\sqrt{\bar{c}}$ is constant ∴ **LCL** &**UCL** for all observations are also constant, giving straight lines $\bar{c} = \frac{23}{5} = 4.6$, z=3

UCL, LCL = 4.6 ± 3$\sqrt{4.6}$ = 11.03 → 11, -1.83 → 0

## u-Chart … 1/2



**Example 5**:

A teacher wishes to monitor and control the class attendance. He records the number of absentees over an academic week and the result is Mon: 5, Tue:3, Wed:3, Thu:2, Fri:4. The number of students on roll on these five days were 40, 42, 42, 38, 38. Develop a control chart.

Explanation: This is a case of defects (absentees) in a unit (class). The sample size (class strength is variable). ∴ Correct Chart is **u**

| Day | Class Strength i.e. Sample Size (n) | Number of Absentees i.e. Defaults/ Defects (d) |
|---|---|---|
| Mon | 40 | 5 |
| Tue | 42 | 3 |
| Wed | 42 | 3 |
| Thu | 38 | 2 |
| Fri | 38 | 4 |

**Cont Limits =** $\bar{u} \pm z\sqrt{\frac{\bar{u}}{n}}$

where
$\bar{u}$ = CL & mean proportional number of defects
$z$ = Quality standard, eg 3 sigma or more, or degree of confidence



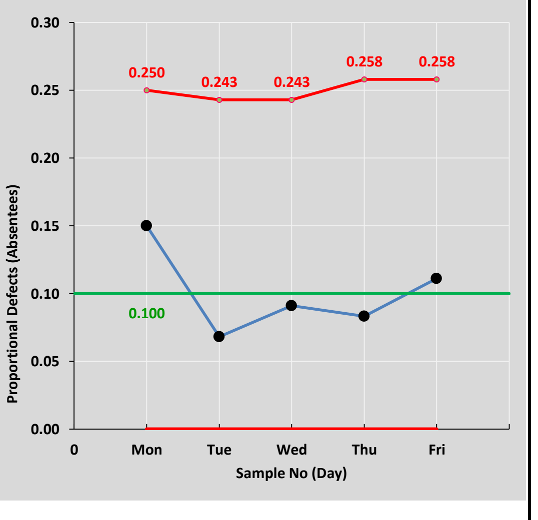
## u-Chart … 2/2





| Day | Sample Size (n) | Defaults/ Defects (d) | Prop Defaults (d/n) | LCL | UCL |
|---|---|---|---|---|---|
| M | 40 | 6 | 0.150 | -0.050 | 0.250 |
| T | 42 | 3 | 0.068 | -0.043 | 0.243 |
| W | 42 | 4 | 0.091 | -0.043 | 0.243 |
| Th | 38 | 3 | 0.083 | -0.058 | 0.258 |
| F | 38 | 4 | 0.111 | -0.058 | 0.258 |
| ∑ | 200 | 20 | | | |
| $\bar{u}$ | 20/200 =**0.10** (10% absentees per day) | | | | |

**Cont Limits =** $\bar{u} \pm z\sqrt{\frac{\bar{u}}{n}}$




In each case (for each day), the sample size $n$ varies. ∴the standard deviation $\sqrt{\frac{\bar{u}}{n}}$, and hence **LCL** &**UCL** for each case vary, and form wiggly lines

## np-Chart … 1/2

**Example 6:** An HOD wishes to monitor & control the class attendance in his Dept. He selects the 8 BBA classes and observes the number of absentees over the period of time. The results are BBA-1: 20, BBA-2: 22, BBA-3: 28, BBA-4: 22, BBA-5: 27, BBA-6: 20, BBA-7: 18 & BBA-8: 21. Assuming that the strength of each class is 40, develop a control chart.

Explanation: This is a case of defectives (classes) with defects (absentees). The sample size (class strength) is fixed ∴ Correct Chart is **np**

$$\text{Cont Limits} = n\bar{p} \pm z\sigma_{np} = n\bar{p} \pm z\sqrt{n\bar{p}(1-\bar{p})}$$

where

$\bar{p}$ = CL & mean of sample proportion defectives = $\dfrac{total\ defectives}{total\ observations}$

$z$ = standard deviation of sample means, eg 3 sigma or more, or degree of confidence

$\sigma_{np}$ = Std Dev of the Average Proportion Defective = $\sqrt{n\bar{p}(1-\bar{p})}$

$n$ = mean sample size

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## np-Chart … 2/2

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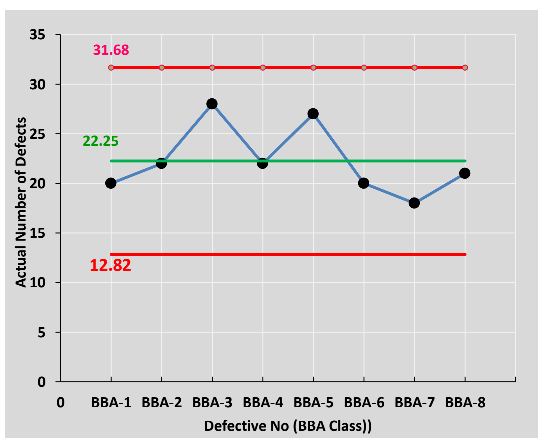
| Defective BBA- | Sample (Class) Size (n) | Def-ects (D) | (Prop Def-ects D/n) | LCL | UCL |
|---|---|---|---|---|---|
| 1 | 40 | 20 | | 12.82 | 31.68 |
| 2 | 40 | 22 | | 12.82 | 31.68 |
| 3 | 40 | 28 | | 12.82 | 31.68 |
| 4 | 40 | 22 | Not | 12.82 | 31.68 |
| 5 | 40 | 27 | Reqd | 12.82 | 31.68 |
| 6 | 40 | 20 | | 12.82 | 31.68 |
| 7 | 40 | 18 | | 12.82 | 31.68 |
| 8 | 40 | 21 | | 12.82 | 31.68 |
| Σ | | 178 | | | |
| $n\bar{p}$ | 178/8 = 22.25 | | | | |
| $\bar{p}$ | 22.25/40 = 0.556 | | | | |




$$\text{Cont Limits} = n\bar{p} \pm z\sigma_{np} = n\bar{p} \pm z\sqrt{n\bar{p}(1-\bar{p})}$$

In each case (for each class), sample size $n$ is the same. ∴ the SD $\sqrt{n\bar{p}(1-\bar{p})}$, and hence **LCL** & **UCL** for each case are also constant, giving straight lines

UCL, LCL = 40x0.556±3$\sqrt{40\text{x}0.556(1-0.556)}$ = 31.68 → 32, 12.82 → 13

## p-Chart … 1/2

**Example 7**: An HOD wishes to monitor & control the class attendance in his Dept. He selects the 8 BBA classes and observes the number of absentees over the period of time. The results are BBA-1: 20, BBA-2: 22, BBA-3: 28, BBA-4: 22, BBA-5: 27, BBA-6: 20, BBA-7: 18 & BBA-8: 21. The class strength was 40, 42, 36, 44, 41, 35, 44 & 43 respectively.

Explanation: This is a case of defectives (classes) with defects (absentees). The sample size (class strength) is variable ∴ Correct Chart is **p**.

**Cont Limits =** $\bar{p} \pm z\sigma_p = \bar{p} \pm z\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$

where

$\bar{p}$ = CL & mean of sample proportion defectives = $\frac{total\ defectives}{total\ observations}$

$z$ = standard deviation of sample means, eg 3 sigma or more, or degree of confidence

$\sigma_p$= Std Dev of the Average Proportion Defective = $\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$

$n$ = mean sample size

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## p-Chart … 2/2

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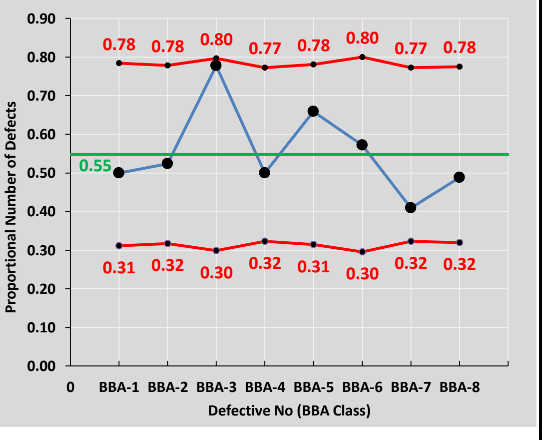
| Defective BBA- | Sample (Class) Size (n) | Def-ects (D) | Prop Def-ects (D/n) | LCL | UCL |
|---|---|---|---|---|---|
| 1 | 40 | 20 | 0.50 | 0.31 | 0.78 |
| 2 | 42 | 22 | 0.52 | 0.32 | 0.78 |
| 3 | 36 | 28 | 0.78 | 0.30 | 0.80 |
| 4 | 44 | 22 | 0.50 | 0.32 | 0.77 |
| 5 | 41 | 27 | 0.66 | 0.31 | 0.78 |
| 6 | 35 | 20 | 0.57 | 0.30 | 0.80 |
| 7 | 44 | 18 | 0.41 | 0.32 | 0.77 |
| 8 | 43 | 21 | 0.49 | 0.32 | 0.78 |
| ∑ | 325 | 178 | | | |
| $\bar{p}$ | 178/325 = 0.55 | | | | |




**Cont Limits =** $\bar{p} \pm z\sigma_p = \bar{p} \pm z\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$

In each case (for each class), the sample size $n$ varies. ∴the SD $\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$, and hence **LCL** & **UCL** for each case vary, and form wiggly lines

## c-Chart … 1/2

**Example 8**:

100 workers are transported to the project site daily in buses. The number of workers missing the buses in the last 10 days has been observed to be 9,8,5,7,9,8,9,4,9,12. Develop a 3-Sigma c-Chart for the defaulting workers

| Day | Number of Workers i.e. Sample Size (n) | Those who missed the buses i.e. Defaults/ Defects (d) |
|---|---|---|
| 1 | 100 | 9 |
| 2 | 100 | 8 |
| 3 | 100 | 5 |
| 4 | 100 | 7 |
| 5 | 100 | 9 |
| 6 | 100 | 8 |
| 7 | 100 | 9 |
| 8 | 100 | 4 |
| 9 | 100 | 9 |
| 10 | 100 | 12 |

**Cont Limits = $\bar{c} \pm z\sqrt{\bar{c}}$**

where
$\bar{c}$ = CL & mean number of defects
$z$ = Quality standard, eg 3 sigma or more, or degree of confidence

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## c-Chart … 2/2

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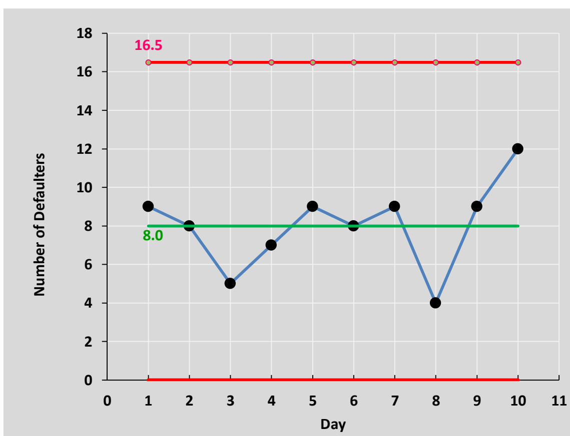
| Day | Defaults/ Defects | Prop Def | LCL | UCL |
|---|---|---|---|---|
| 1 | 9 | | -0.5 | 16.5 |
| 2 | 8 | | -0.5 | 16.5 |
| 3 | 5 | | -0.5 | 16.5 |
| 4 | 7 | | -0.5 | 16.5 |
| 5 | 9 | Not | -0.5 | 16.5 |
| 6 | 8 | Reqd | -0.5 | 16.5 |
| 7 | 9 | | -0.5 | 16.5 |
| 8 | 4 | | -0.5 | 16.5 |
| 9 | 9 | | -0.5 | 16.5 |
| 10 | 12 | | -0.5 | 16.5 |
| ∑ | 80 | | | |
| $\bar{c}$ | 80/10 =8.0 per day | | | |




**Cont Limits = $\bar{c} \pm z\sqrt{\bar{c}}$**

Here, the SD $\sqrt{\bar{c}}$ is constant $\therefore$ **LCL** &**UCL** for all observations are also constant, giving straight lines $\bar{c} = \frac{80}{10} = 8.0$, z=3

UCL, LCL = 8.0 $\pm$ 3$\sqrt{8.0}$ = 16.5 → 17, -0.5 → 0

## u-Chart … 1/2

**Example 9**:

A construction company transports its workers to project site in buses. The number of workers on the company's register over the last 10 days, and those who missed the buses are as tabulated. Develop a 3-Sigma u-Chart for the defaulting workers



| Day | Number of Workers i.e. Sample Size (n) | Those who missed the buses i.e. Defaults/ Defects (d) |
|---|---|---|
| 1 | 98 | 9 |
| 2 | 100 | 8 |
| 3 | 100 | 5 |
| 4 | 102 | 7 |
| 5 | 100 | 9 |
| 6 | 99 | 8 |
| 7 | 99 | 9 |
| 8 | 100 | 4 |
| 9 | 100 | 9 |
| 10 | 102 | 12 |

**Cont Limits =** $\bar{u} \pm z\sqrt{\frac{\bar{u}}{n}}$

where

$\bar{u}$ = CL & mean proportional number of defects

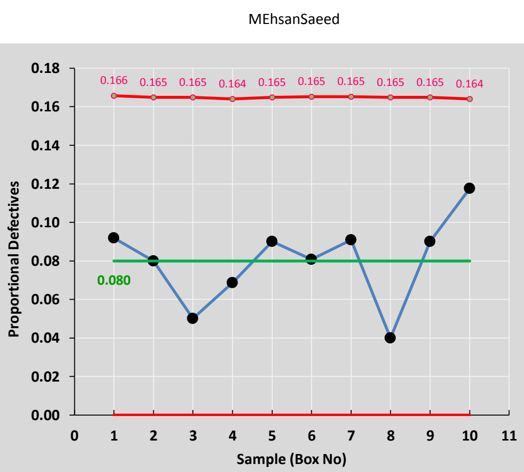
$z$ = Quality standard, eg 3 sigma or more, or degree of confidence





## u-Chart … 2/2

| Day | Sample Size (n) | Defaults/ Defects (d) | Prop Defaults (d/n) | LCL | UCL |
|---|---|---|---|---|---|
| 1 | 98 | 9 | 0.092 | -0.006 | 0.166 |
| 2 | 100 | 8 | 0.080 | -0.005 | 0.165 |
| 3 | 100 | 5 | 0.050 | -0.005 | 0.165 |
| 4 | 102 | 7 | 0.069 | -0.004 | 0.164 |
| 5 | 100 | 9 | 0.090 | -0.005 | 0.165 |
| 6 | 99 | 8 | 0.081 | -0.005 | 0.165 |
| 7 | 99 | 9 | 0.091 | -0.005 | 0.165 |
| 8 | 100 | 4 | 0.040 | -0.005 | 0.165 |
| 9 | 100 | 9 | 0.090 | -0.005 | 0.165 |
| 10 | 102 | 12 | 0.118 | -0.004 | 0.164 |
| ∑ | 1,000 | 80 | | | |
| $\bar{u}$ | 80/1,000 =**0.08** | | | | |




**Cont Limits =** $\bar{u} \pm z\sqrt{\frac{\bar{u}}{n}}$

In each case (for each day), the sample size $n$ varies. ∴ the standard deviation $\sqrt{\frac{\bar{u}}{n}}$, and hence **LCL** &**UCL** for each case vary, and form wiggly lines

## np-Chart … 1/2

**Example 10:**
On a large construction project, 10 boxes of electrical switches have arrived. Each box has 1,000 switches. Randomly, the procurement manager picks up 20 switches each from the 10 boxes. He finds 3, 3, 4, 2, 1, 3, 2, 3, 2 & 1 switches defective, in the ten boxes. Develop a 3-sigma np-Chart for the sampling done.

**Cont Limits =** $n\bar{p} \pm z\sigma_{np} = n\bar{p} \pm z\sqrt{n\bar{p}(1-\bar{p})}$

where

$\bar{p}$ = CL & mean of sample proportion defectives = $\frac{total\ defectives}{total\ observations}$

$z$ = standard deviation of sample means, eg 3 sigma or more, or degree of confidence

$\sigma_{np}$= Std Dev of the Average Proportion Defective = $\sqrt{n\bar{p}(1-\bar{p})}$

$n$ = mean sample size

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## np-Chart … 2/2

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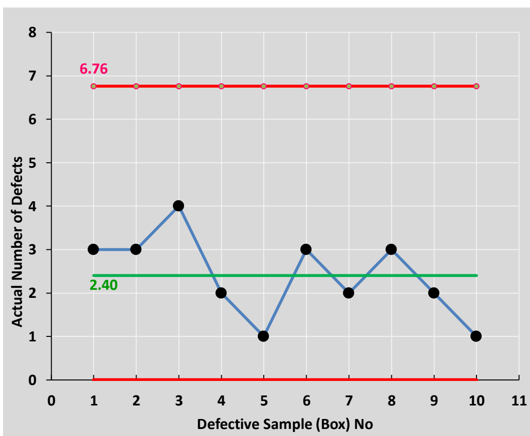
| Def Box No | Sample Size (n) | Defects (d) | (Prop defects (d/n)) | LCL | UCL |
|---|---|---|---|---|---|
| 1 | 20 | 3 | | -1.96 | 6.76 |
| 2 | 20 | 3 | | -1.96 | 6.76 |
| 3 | 20 | 4 | | -1.96 | 6.76 |
| 4 | 20 | 2 | | -1.96 | 6.76 |
| 5 | 20 | 1 | Not | -1.96 | 6.76 |
| 6 | 20 | 3 | Reqd | -1.96 | 6.76 |
| 7 | 20 | 2 | | -1.96 | 6.76 |
| 8 | 20 | 3 | | -1.96 | 6.76 |
| 9 | 20 | 2 | | -1.96 | 6.76 |
| 10 | 20 | 1 | | -1.96 | 6.76 |
| $\sum$ | | 24 | | | |
| $n\bar{p}$ | 24/10 = 2.4 | | | | |
| $\bar{p}$ | 2.4/20 = 0.12 | | | | |




**Cont Limits =** $n\bar{p} \pm z\sigma_{np}$ **=** $n\bar{p} \pm z\sqrt{n\bar{p}(1-\bar{p})}$

In each case (for each box), the sample size $n$ is the same. ∴the SD $\sqrt{n\bar{p}(1-\bar{p})}$, and hence **LCL** &**UCL** for each case are also constant, giving straight lines

UCL, LCL = 20x0.12±3$\sqrt{20\text{x}0.12(1-0.12)}$ = 6.76 → 7, -1.96 → 0

## p-Chart … 1/2

**Example 11:**
On a large construction project, 10 boxes of electrical switches have arrived. Each box has 1,000 switches. Randomly, the procurement manager picks up 22, 20, 18, 18, 18, 20, 15, 18, 18 & 20 switches from the 10 boxes. He finds 3, 2, 1, 2, 1, 3, 2, 1, 2 & 3 switches defective, respectively, in the ten boxes. Develop a 3-sigma p-Chart for the sampling done.

**Cont Limits =** $\overline{p} \pm z\sigma_p = \overline{p} \pm z\sqrt{\frac{\overline{p}(1-\overline{p})}{n}}$

where

$\overline{p}$ = CL & mean of sample proportion defectives = $\frac{total\ defectives}{total\ observations}$

$z$ = standard deviation of sample means, eg 3 sigma or more, or degree of confidence

$\sigma_p$= Std Dev of the Average Proportion Defective = $\sqrt{\frac{\overline{p}(1-\overline{p})}{n}}$

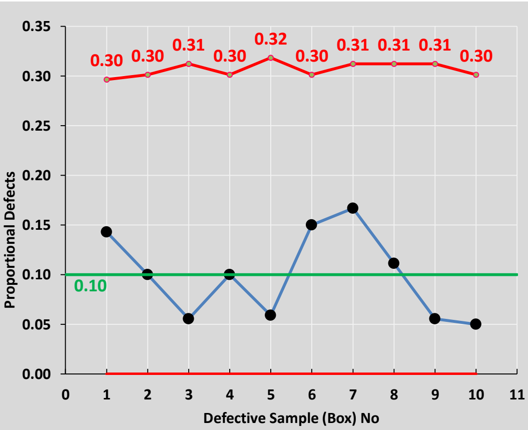
$n$ = mean sample size



## p-Chart … 2/2



| Def Box No | Sample Size (n) | Defects (d) | Prop Def-ects (d/n) | LCL | UCL |
|---|---|---|---|---|---|
| 1 | 21 | 3 | 0.14 | -0.10 | 0.30 |
| 2 | 20 | 2 | 0.10 | -0.10 | 0.30 |
| 3 | 18 | 1 | 0.06 | -0.11 | 0.31 |
| 4 | 20 | 2 | 0.10 | -0.10 | 0.30 |
| 5 | 17 | 1 | 0.06 | -0.12 | 0.32 |
| 6 | 20 | 3 | 0.15 | -0.10 | 0.30 |
| 7 | 18 | 3 | 0.17 | -0.11 | 0.31 |
| 8 | 18 | 2 | 0.11 | -0.11 | 0.31 |
| 9 | 18 | 1 | 0.06 | -0.11 | 0.31 |
| 10 | 20 | 1 | 0.05 | -0.10 | 0.30 |
| ∑ | 190 | 19 | | | |
| $\overline{p}$ | 19/190=0.100 | | | | |




**Cont Limits =** $\overline{p} \pm z\sigma_p = \overline{p} \pm z\sqrt{\frac{\overline{p}(1-\overline{p})}{n}}$

In each case (for each box), the sample size $n$ varies. ∴the SD $\sqrt{\frac{\overline{p}(1-\overline{p})}{n}}$, and hence **LCL** & **UCL** for each case vary, and form wiggly lines